Active Suspension Controller Design Using MPC with Preview Information

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The object is to develope a control law for an active suspension for the purpose of the improvement of ride characterisitcs. For this purpose the Model Predicitive Control methodology is applied and it is assumed that the preview information of the oncoming road disturbance is available. It is very important to consider the physical limits on the suspension travel for the vehicle running over a rough road. Thus the limits of suspension travel are accounted. Numerical simulations with the same model on the same road show that the MPC controller achieves great improvement for the ride qualities of a vehicle.

Key Words: Active Suspension, Model Predictive Control(MPC), Preview Information, Constrained Optimal Control

1. Introduction

Passive vehicle suspensions are based on a trade-off between conflicing requirements. To obtain a high ride quality, active/semi-active suspensions were proposed. Especially active suspension control with preview strategies have been shown by numerous researchers to be effective in improving the ride qualities of a vehicle over any other suspensions (Sharp and Pilbeam, 1993; Thompson, *et al.*, 1989; Tomizuka, 1976). However, extreme conditions encountered by off-road vehicles driven over rough terrain, demand additional features from these control strategies. The considerations of the physical limits on the suspension travel become significant for these situations. Harsh bumps might cause the suspension to hit the physical stops known as "bump-stopper". The impact produces a significant jerk on the car chassis and introduces undesired accelerations into the system and degrades the ride characterisitics of the vehicle.

The main goal of this study is to design and evaluate an active suspension controller which maximizes the ride comfort of a vehicle by using road preview information and by considering the physical limits on the suspension travel.

Some of the prevalent techniques used for the design of semi-active or active suspension controllers are sky-hook damping, optimal LQR and optimal LQR with preview (Park and Koo, 1994 ; Kim and Yoon, 1994 ; Hac, 1992). None of these controllers has any provisions for taking into explicit consideration constraints on any of the states. There were some researches about constrained semi-active suspension control (Cho and Yi, 1997 ; Aa, et al., 1997). These researches are very successful on smooth road but with semi-acitve suspension the performance is not satisfied over rough terrain. For vehicles over rough terrain active suspension is more appropriate. The Model Predictive Control (MPC) frame work (Clark, 1994 ; Mehra, et al., 1982) promises to be a suitable tool for this application since it allows the explicit considerations of the physical limits on suspension travel in the controller design. Furthermore, this framework offers the ability to switch suspension spring stiffness based on the predicted suspension travel.

The predicitive control approach uses the Output prediction and a receding-horizon approach. It uses a predictor to forecast the

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output over a time horizon and determines the future control over the horizon by minimizing the cost function. Of the future control determined only the first control is used because of the receding horizon approach. The same steps are repeated for the next sampling instant. The constrained Predicitive Control problem can be recast as constrained Quadratic Problem.

A quarter car suspension model is described in Sec. 2. In Sec. 3 the control law of the active suspension with preview information is described and the control strategy is presented. Sec. 4 presents the numerical simulation results and finally conclusions are drawn in Sec. 5.

2. Suspension Model

Consider a quarter car suspension model in Fig. 1. In Fig. 1 z_s, z_u and z_r are the vertical displacements of sprung mass, unsprung mass and ground, respectively. With state vector $x=[z_u - z_r \;\; \dot{z}_u \;\; z_s - z_u \;\; \dot{z}_s]^T$, the state equations for the model may be written in matrix form,

$$\dot{x}(t) = Ax(t) + B_u u(t) + B_v v(t) \qquad (1)$$
$$y(t) = Cx(t) + Du(t)$$

where x_1, x_3 are tire and suspension deflections from equilibrium position, x_2, x_4 are unsprung and sprung mass velocities, u is control input, f_a, and v is road disturbance, $\dot{z}_r$. State space euqation (1) can be reconstructed to the discretized state space equation for digital control such as,

$$\dot{x}(k+1) = A_d x(k) + B_{ud} u(k) + B_{vd} v(k) \qquad (2)$$
$$y(k) = Cx(k) + Du(k)$$

Three horizons, N, P, and Nc, involved in the Predictive Control formulation are adopted as in Fig. 2. It will be assumed that the road disturbance, v, is known accurately through the preview window, P, i.e. $v(k) \cdots v(k+P-1)$ are known accurately at time step k using road preview sensor. The accuracy of the road preview sensor is out of scope of this paper. Control inputs are permitted to vary only within control window, N, and between control window and preview window in Fig. 2 control inputs are held

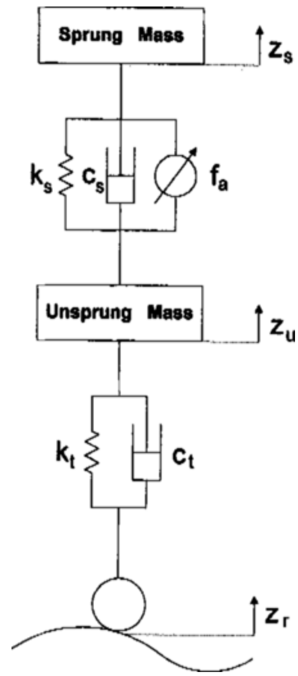


Fig. 1 Quarter car suspension model.

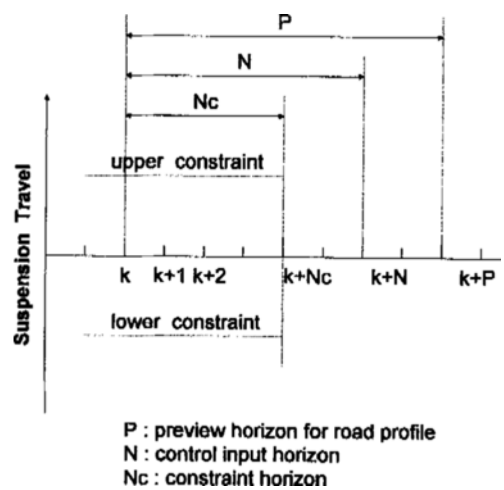


Fig. 2 Predicitve control horizon.

constant, i.e.

$$u(k+N) = u(k+N+1) = \cdots = u(k+P)$$

Output y is composed of suspension travel, tire diflection, and sprung mass acceleration.

3. Control Law Formulation

3.1 Performance index

The controller has to minimize the quadratic performance index,

$$J=\sum_{i=1}^{P}y^T(k+i)\bar{Q}(k+i)y(k+i)$$
$$+\sum_{i=0}^{N}u^T(k+i)\bar{R}(k+i)u(k+i) \quad (3)$$

where sysmmetric and positivie definite matrices $\bar{Q}$ and $\bar{R}$ are used to put emphasis on individual performance quantities in relation with the input effort.

State and output constraints may be incorporated by defining an constraint output vector such as,

$$y_c(k)=C_cx(k)+D_cu(k) \quad (4)$$

The constraints over a constraint horizon Nc can be expressed as,

$$low_c \le y_c(k+i) \le up_c \quad i=1, \cdots, N_c$$

The performance index as in Eq. (3) can be written in the equivalent vector space form,

$$J=\hat{y}^TQ\hat{y}+\hat{u}^TR\hat{u} \quad (5)$$

with constraints,

$$L_c \le \hat{y}_c \le U_c \quad (6)$$

where

$$\hat{y}=[y(k+1)\cdots y(k+P)]^T$$
$$\hat{u}=[u(k)\cdots u(k+N)]^T$$
$$\hat{v}=[v(k)\cdots v(k+P-1)]^T$$
$$\hat{y}_c=[y_c(k+1)\cdots y_c(k+N_c)]^T$$
$$L_c=[low_c\cdots low_c]^T$$
$$U_c=[up_c\cdots up_c]^T$$
$$Q=diag(\bar{Q}(k+1)\cdots\bar{Q}(k+P))$$
$$R=diag(\bar{R}(k)\cdots\bar{R}(k+N))$$

Actuator has its limit and it will work as a constraint, such that,

$$u_{\min} \le u \le u_{\max}.$$

3.2 Control law

The MPC controller uses an output predictor and a receding horizon approach. It uses a predictive model to predict the output over a finite time horizon and determine the future input control over the horizon that minimizes a performance index in Eq. (5). Among the future control sequence determined, only the first control is applied to the system because of the receding horizon approach and the same steps are repeated for the next sampling instant.

From Eq. (2), output predictor is given by,

$$\hat{y}=\Lambda x(k)+\Gamma_u\hat{u}+\Gamma_v\hat{v} \quad (7)$$

where

$$\Lambda=\begin{bmatrix} CA_d \\ CA_d^2 \\ \vdots \\ CA_d^N \\ \vdots \\ CA_d^P \end{bmatrix}$$

$$\Gamma_u=\begin{bmatrix} CB_{ud} & D & & \\ CA_dB_{ud} & CB_{ud} & & \\ \vdots & \vdots & & \\ CA_d^NB_{ud} & CA_d^{N-1}B_{ud} & \cdots & D \\ CA_d^{N+1}B_{ud} & CA_d^NB_{ud} & \cdots & D+CB_{ud} \\ \vdots & \vdots & \vdots & \vdots \\ CA_d^{P-1}B_{ud} & CA_d^{P-2}B_{ud} & \cdots & (D+\sum_{i=1}^{P-N}CA_d^{i-1}B_{ud}) \end{bmatrix}$$

$$\Gamma_u=\begin{bmatrix} CB_{vd} & & & & \\ CA_dB_{vd} & CB_{vd} & & & \\ \vdots & \vdots & & & \\ CA_d^NB_{vd} & CA_d^{N-1}B_{vd} & \cdots & CB_{vd} & \\ \vdots & \vdots & \vdots & \vdots & \\ CA_d^{P-1}B_{vd} & CA_d^{P-2}B_{vd} & \cdots & CA_d^{P-N-2}B_{vd} & \cdots\ CB_{vd} \end{bmatrix}$$

In a similar manner, output constraint predictor are obtained.

$$\hat{y}_c=\Lambda_cx(k)+\Gamma_{uc}\hat{u}+\Gamma_{vc}\hat{v} \quad (8)$$

Replacing the predictor equation given in Eq. (7) for output in the performance index given in Eq. (5), it becomes,

$$J=(\Lambda x(k)+\Gamma_u\hat{u}+\Gamma_v\hat{v})^TQ(\Lambda x(k)$$
$$+\Gamma_u\hat{u}+\Gamma_v\hat{v})+\hat{u}^TR\hat{u} \quad (9)$$

Because the controller should minimize the performance index by adjusting control input u with the knowledge of current state vector $x(k)$, in Eq. (9) only the terms containing $\hat{u}$ are important in the minimization procedure. So the performance index may be reformed as,

$$J=\frac{1}{2}\bar{u}^T(\Gamma_u^T Q\Gamma_u+R)\bar{u}+x^T\Lambda^T Q\Gamma_u\bar{u}+\bar{v}^T\Gamma_v^T Q\Gamma_u\bar{u} \tag{10}$$

Substituting Eq. (8) into Eq. (6), the constraints can be expressed as,

$$L_c-\Lambda_c x(k)-\Gamma_{vc}\bar{v}\leq\Gamma_{uc}\bar{u}\leq U_c-\Lambda_c x(k)-\Gamma_{vc}\bar{v} \tag{11}$$

Eq. (11) can be written in matrix form such that,

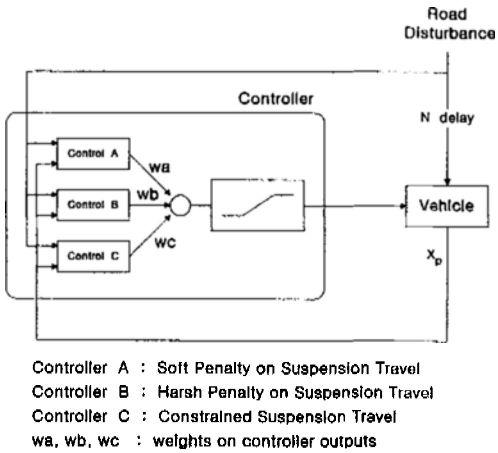
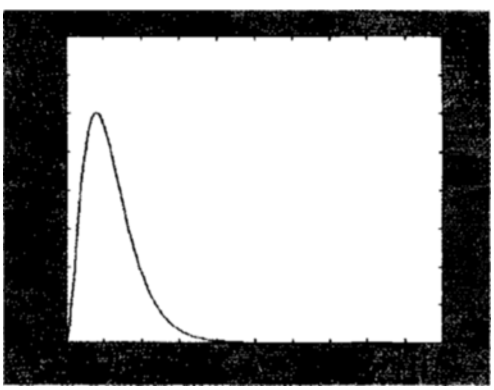


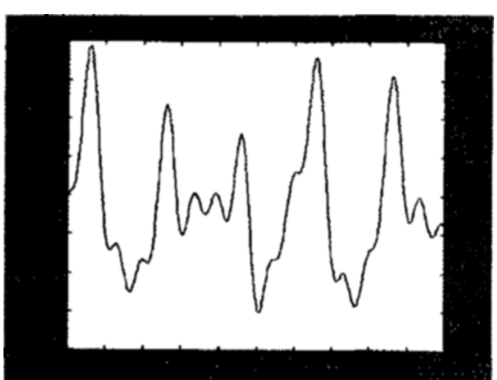
Fig. 3 Gain control.

Table 1 Quarter car paramenter of BASR.

Description	Symbol	Value
Sprung Mass	m_s	285.3kg
Unsprung Mass	m_u	59.5kg
Suspension Stiffness	k_s	16,812.0N/m
Suspension Damping	c_s	1000.0N/m/sec
Tire Stiffness	k_t	190,000.0N/m
Tire Damping	c_t	15.0N/m/sec
Suspension Limit	low_c, up_c	0.056m

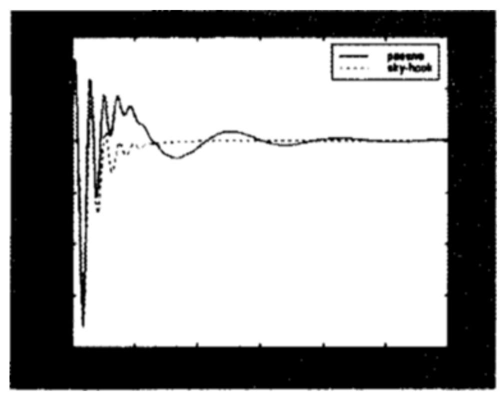


(a) Rounded bump profile

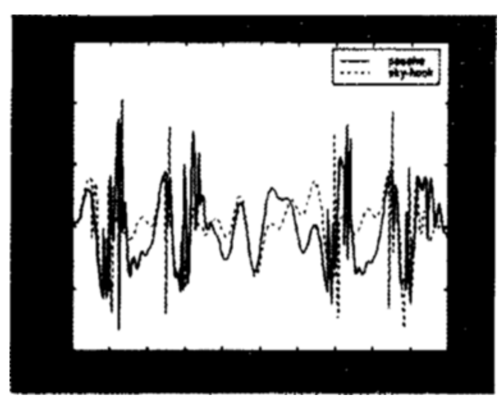


(b) Random road profile

Fig. 4 Road profiles.



(a) On rounded pulse



(b) On random road

Fig. 5 Responses of passive and sky-hook controller.

$$\begin{bmatrix} \Gamma_{uc} \\ -\Gamma_{uc} \end{bmatrix} \hat{u} \le \begin{bmatrix} U_c \\ -L_c \end{bmatrix} + \begin{bmatrix} -\Lambda_c & -\Gamma_{vc} \\ \Lambda_c & \Gamma_{vc} \end{bmatrix} \begin{bmatrix} x(k) \\ \hat{v} \end{bmatrix} \quad (12)$$

Then controler shoud solve a QP optimization problem as follows,

$$\min_u \frac{1}{2} \hat{u}^T (\Gamma_u^T Q \Gamma_u + R) \hat{u} + (x^T \Lambda^T Q \Gamma_u + \hat{v}^T \Gamma_u^T Q \Gamma_u) \hat{u}$$

subject to two kinds of constraints, suspension travel limits and actuating force limit as follows,

$$\begin{bmatrix} \Gamma_{uc} \\ -\Gamma_{uc} \end{bmatrix} \hat{u} \le \begin{bmatrix} U_c \\ -L_c \end{bmatrix} + \begin{bmatrix} -\Lambda_c & -\Gamma_{vc} \\ \Lambda_c & \Gamma_{vc} \end{bmatrix} \begin{bmatrix} x(k) \\ \hat{v} \end{bmatrix}$$

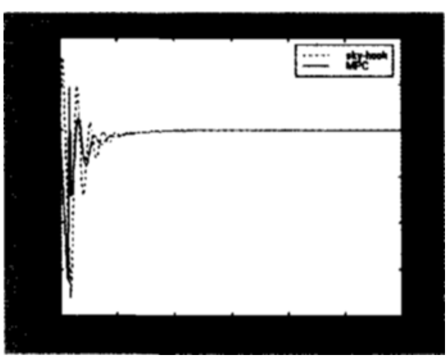
$$u_{\min} \le u \le u_{\max}$$

3.3 Gain Schedule

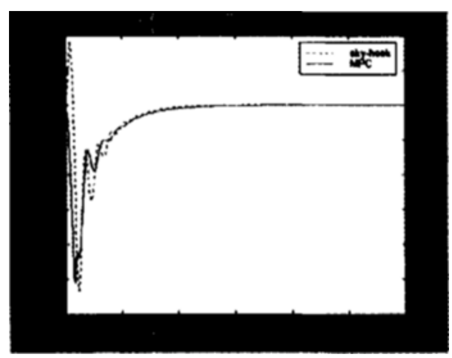
The MPC formulation can allow the systems to have the additional feature of scheduling the control gains based on the predicted suspension travel.

The full range of the suspension travel limited by the physical stops of the suspecsion, which are called "bump stopper", was divided into three regions. Region A is a small suspension deflection region within soft limits where the reduction of sprung mass acceleration is more important, and region B is a large suspρnsion deflection region between soft and hard limits where the reduction of suspension travel is important as well as the reduction of sprung mass acceleration. Region C is bump stopper contact region where suspension travel is constrained and the reduction of suspension travel is urgent. Three controllers, A, B and C, were designed as in Fig. 3. Controller A is an unconstrained MPC with a soft penality on suspension travel, Controller B is an unconstrained MPC with a stiff penalty on suspension travel, and Controller C is a constrained MPC.

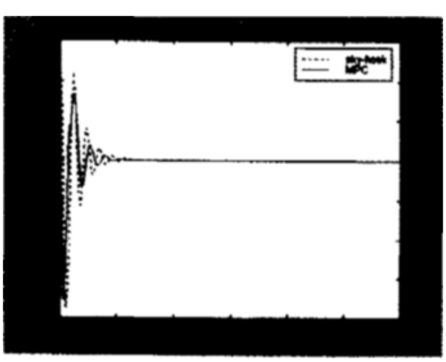
Using the output predictor controller calcuates the future suspension travel over the constraint horizone, Nc. If the predicted suspension travel is within the Region A or Region B, controller uses Controller A or Controller B, respectively. If the



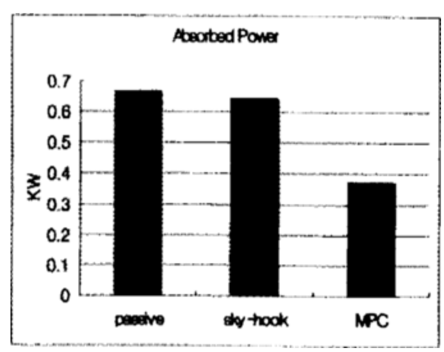
(a) Sprung mass acceleration



(b) Suspension deflection

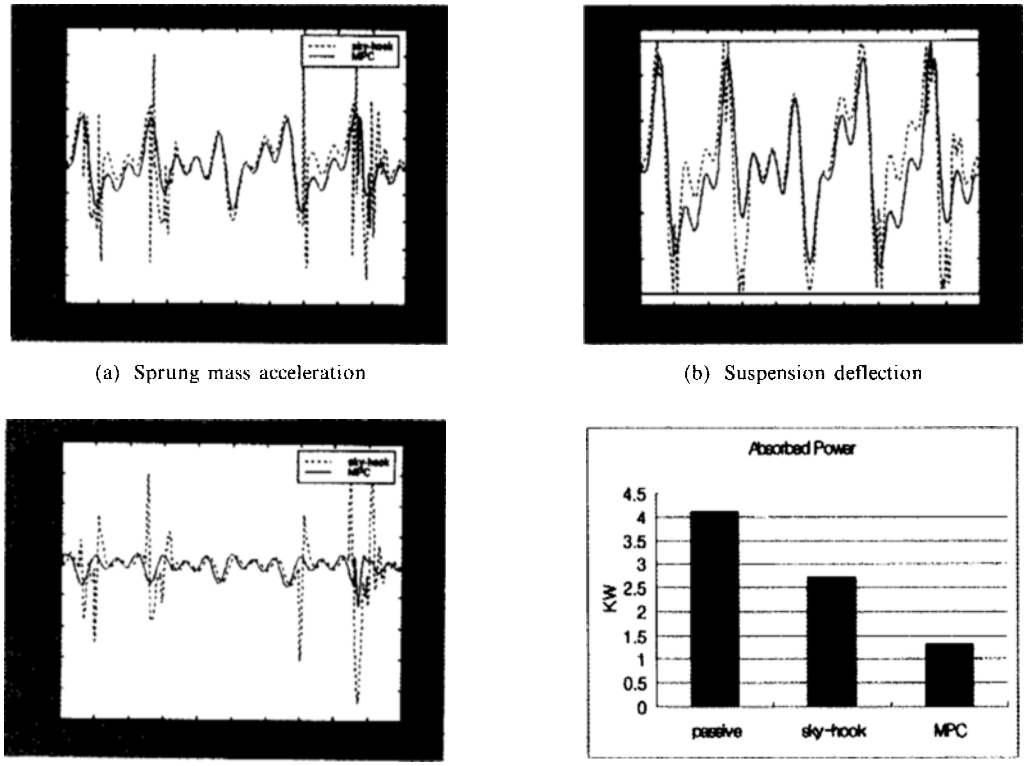


(c) Tire deflection



(d) Absorbed power

Fig. 6 Reponses of rounded pulse



(a) Sprung mass acceleration

(b) Suspension deflection

(c) Tire deflection

(d) Absorbed power

Fig. 7 Reponses on random road

predicted suspension travel exceeds the limit, controller used Controller C.

4. Numerical Simulation and Results

Table 1 shows the suspension parameters used for simulations which are those of the Berkeley Active Suspension Rig(BASR). For comparative purposes, a passive suspension and a sky-hook controller whose gain factors were selected for optimized performances were simulated too. For the numerical simulation of suspension control, two types of road profile were generated, rounded pulse and pseudo-random road as in Fig. 4.

Rounded pulse is used to evaluate the performance of the suspension for deterministic road disturbances. These rounded pulses are described as a function of the horizontal vehicle position s, by the equation,

$$w = w_{max}\frac{e^2}{4}\left(2\pi\frac{s}{l_d}\right)^2\exp\left(-2\pi\frac{s}{l_d}\right)$$

where pulse shape is determined by w_{max} and characteristic length l_d. In this study $w_{max}=0.06$ [m] and $l_d=1$[m] were used. The velocity of the vehicle is 45[km/h], which means t_d is about 0.08 [s] and it is 8 times of sampling rate, $t_p=0.01$[s]. Pseudo-random road is composed of various sinusoidal funcitons.

In Fig. 5 sprung mass accelerations of passive suspension and sky-hook controller over rounded pulse and pseudo-random road are shown. On rounded pulse sprung mass acceleration of sky-hook controller was damped much faster than that of passive suspension, and on random road jerking took place in both of suspensions but it was more serious in passive suspension. It shows sky-hook controller is superior to passive suspension.

In Fig. 6 the reponses of MPC and sky-hook controller over the rounded pulse are compared. It could be observed that sprung mass acceleration of the MPC is smaller and damped much faster than that of sky-hook controller, and that suspension deflection of MPC is somewhat smaller too. As a result the MPC can reduce the absorbed power by a driver by about a half. This means MPC can offer good ride comfortness. Tire deflection of the MPC is somewhat smaller than that of sky-hook controller. This means MPC can produce better roadholding. And good roadholding produces good handling performances. Suspension deflections of both suspensions are limited between the bump-stopper ranges over this bump.

In Fig. 7 the responses over the pseudo-random road are shown. In this figure it can be seen that suspension deflection of the sky-hook controller exceeds the limits, and consequently bump-stopper hits chassis. This impact produces a significant jerk on the car chassis and introduces undesired accelerations into the system and degrades the ride characteristics of the vechicle. But this situation does not make happen in the MPC. As a result, the shock absorbing performance of the MPC is much better than that of the sky-hook controller. Tire deflection is much smaller than that of sky-hook controller. Consequently, it can be seen that the MPC improves ride and handling performance so much.

5. Conclusion

In this paper, an MPC active suspension controller that incorporates preview information and a constraint on the suspension travel was designed. The MPC controller grealty enhances the ride characteristics compared with the passive suspension and the optimized sky-hook controller. With the preview information of road and the consideration of the suspension travel constraint, the MPC manages to compensate for the trade-off between suspension travel and chassis without significant degrading the ride characterisitcs of the vehicle system.

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